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Preface

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Preface

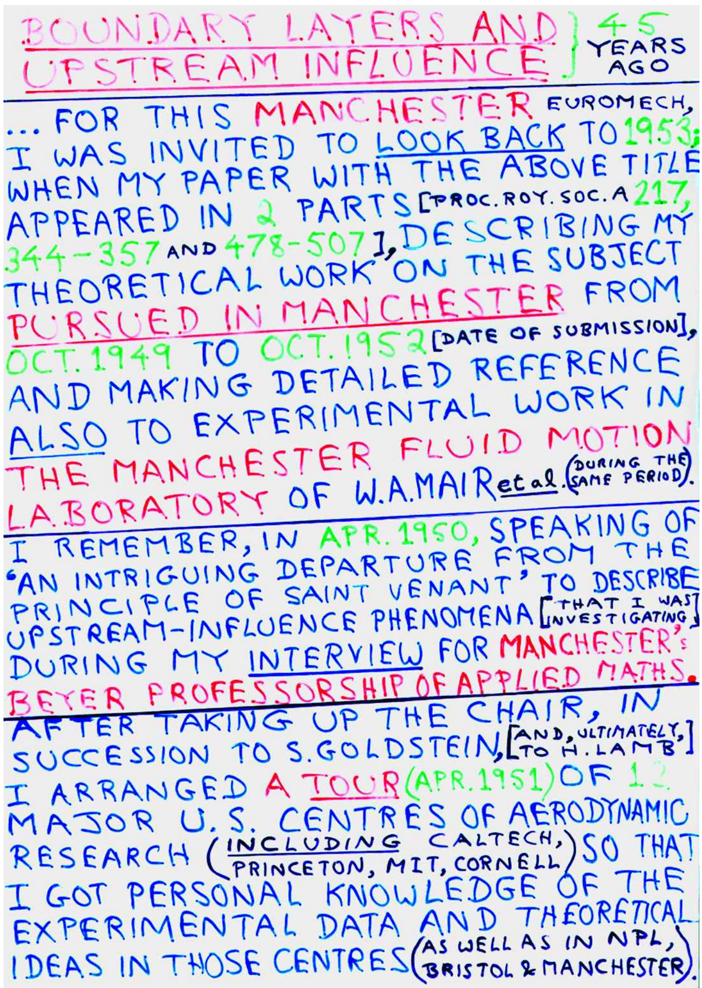
The papers contained in this Theme Issue are based on a selection of contributions made at the EUROMECH 384 Colloquium on Steady and Unsteady Separated Flows, held in Manchester in July 1998. The purpose of the meeting was to bring together experimentalists, theoreticians and computationalists, to enhance the development and understanding of all aspects of steady and unsteady separated flows. These are important both from the fundamental and practical points of view, because such flow phenomena often affect entire flow fields and the aerodynamic forces acting on bodies in these flows. A wide range of aspects of flow separation was addressed, including steady and unsteady flows, two- and three-dimensional flows, laminar and turbulent, small- and large-scale, incompressible and compressible, external and internal, boundary layers and shear layers, stall and bubbles; a number of the participants were also interested in stability/transition effects, since much of the flow physics is inherently connected with the separation process. The articles selected reflect the interdisciplinary and broad spectrum of issues addressed at the meeting.

The previous major international meeting dedicated to flow separation was the IUTAM symposium in Novosibirsk in 1990, and therefore it is some years since a proceedings devoted to the topic has been published, in which time there have been a number of important developments. Also, appropriately, 1998 marked the 50th anniversary of the landmark paper by Sydney Goldstein on the nature of the singularity encountered in many boundary-layer solutions at the point of flow separation.

Sir James Lighthill, FRS, was an invited keynote speaker at the colloquium, but tragically died just a few days after the meeting. As usual, Sir James played a full and active role at the meeting, sitting at the front of the meeting room, interjecting with decisive questions. Sir James had agreed to prepare a manuscript, based on his presentation, and, indeed, had started preparation of his paper prior to his death. We are indebted to Professor Frank Smith for generously completing Sir James's contribution. Additionally, Sir James's presentational transparencies were legendary: full of colour and packed with information, with every square inch fully utilized. Again, thanks to Frank Smith we are able to reproduce these in this Theme Issue. The assistance of the Lighthill family is also very gratefully acknowledged. We dedicate this issue to the memory of Sir James Lighthill.

P. W. Duck
A. I. Ruban

Sir James Lighthill's transparencies



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On boundary layers and upstream influence.
I. A comparison between subsonic and supersonic flows

By M. J. Lighthill*

Department of Mathematics, University of Manchester

(*Communicated by M. H. A. Newman, F.R.S.—Received* 25 *October* 1952)

It is pointed out that there are two separate mechanisms for upstream influence through the boundary layer in supersonic flow, and that one of these (that involving separation) operates also in subsonic flow. A quantitative theory of subsonic flow up a step is given to illustrate this. The main differences between the subsonic and supersonic flows are as follows:

(i) The boundaries of dead-air regions are nearly straight in supersonic flow but are usually highly curved in subsonic flow.

(ii) Separation (whether of the laminar or turbulent layer) occurs at a much lower pressure coefficient in supersonic flow; this is only slightly due to the fact that the fluid nearest the wall is then lighter and so more easily brought to rest; it is due much more to the relative suddenness of the pressure rise ahead of the dead-air region.

(iii) However, for a given pressure coefficient in the dead-air region, the distance of upstream influence is somewhat greater in the subsonic flow, except at the highest pressures.

A qualitative discussion of the second mechanism of upstream influence, in supersonic flow, is given; for a quantitative theory of this see part II (Lighthill 1953).

1. Introduction: mechanisms of upstream influence

38

In the well-known inviscid theory of supersonic flow, a disturbance at a point can have an upstream influence only if it is so strongly compressive as to reduce the local fluid speed below that of sound. But it is now many years since Ferri (1939) found that disturbances can have an upstream influence, through the agency of the boundary layer, when on the inviscid theory they would have none.

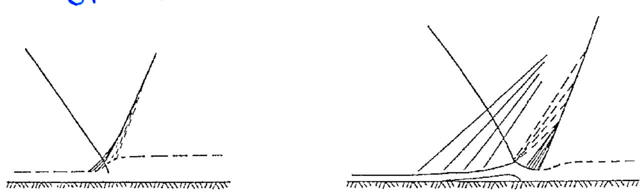
Now that a large body of experimental evidence is available (Liepmann 1946; CALTECH
Liepmann, Ashkenas & Cole 1947; Liepmann, Roshko & Dhawan 1949; Ackeret, ZURICH
NPL Feldmann & Rott 1946; Fage & Sargent 1947; Barry, Shapiro & Neumann 1950; MIT
Holder & North 1950; Gadd & Holder 1952; Bardsley & Mair 1951; Mair 1952; MANCHESTER
PRINCETON Bogdonoff & Solarski 1951; Johannesen 1952), and that considerable effort has
been put into working out the consequences of various theories on the subject
(Oswatitsch & Wieghardt 1941; Howarth 1948; Lees 1949; Lees & Crocco 1952; BRISTOL
Tsien & Finston 1949; Lighthill 1950; Robinson 1950; Stewartson 1951; Kuo 1951), CORNELL
it has become clear that two separate mechanisms exist, by means of which the boundary layer acts to transmit the influence of a disturbance upstream. These are as follows:

(i) (Oswatitsch & Wieghardt 1941.) A disturbance leading to a positive pressure gradient causes the boundary layer to thicken; similarly, one leading to a negative gradient causes it to thin. In either case it must begin to curve slightly upstream, and this curvature itself produces (as the simple linear two-dimensional theory of supersonic flow makes plain) a pressure gradient in the *same* sense slightly upstream.

* Elected F.R.S. on 19 March 1953.

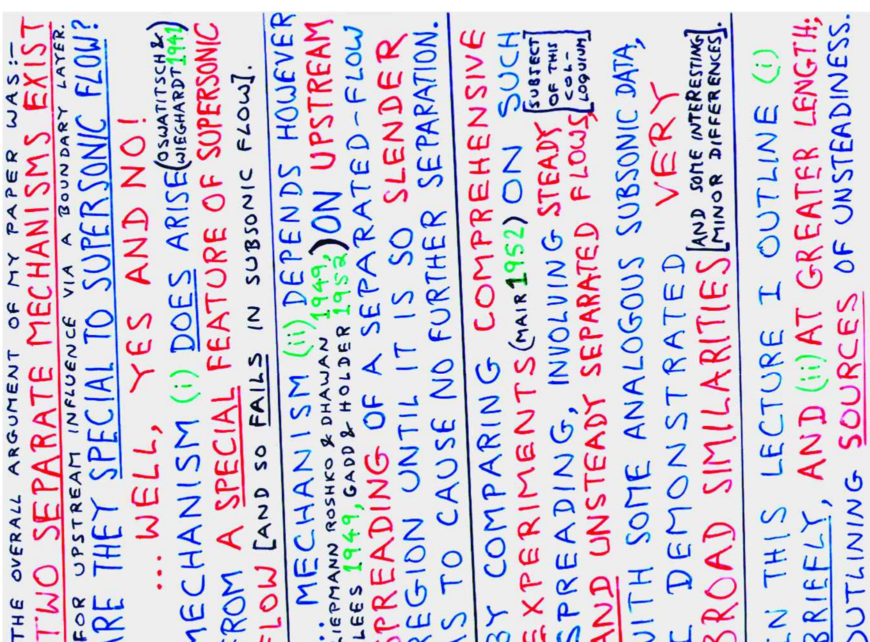
[344]

AS ILLUSTRATED IN THESE SKETCHES OF 'TYPICAL INTERACTIONS':-

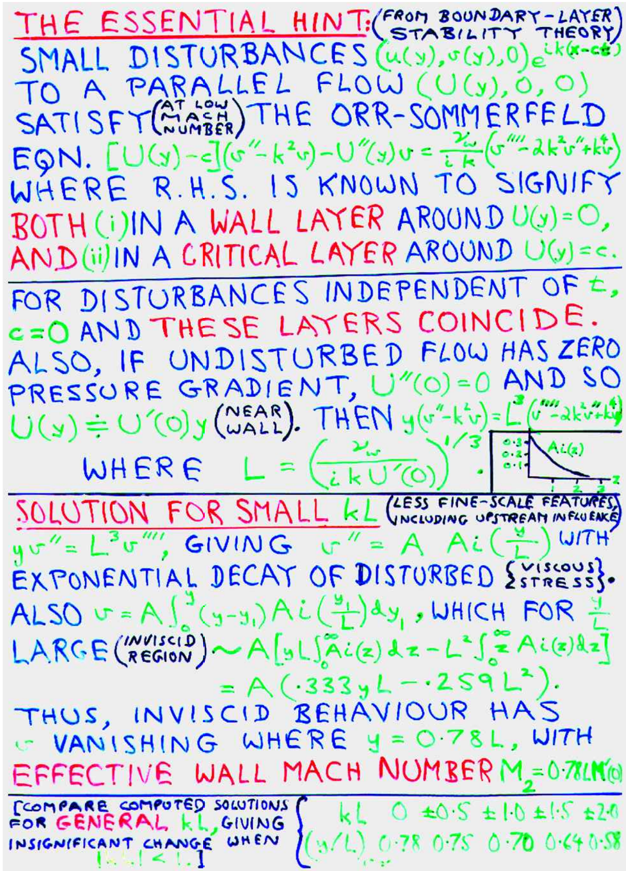


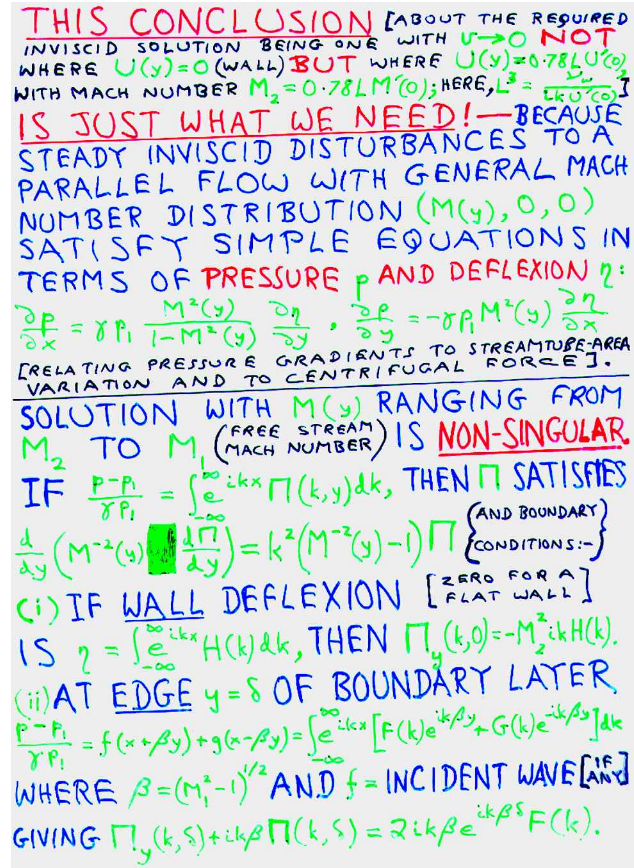
Turbulent boundary layer. Laminar boundary layer.

Reflection patterns.


THE OVERALL ARGUMENT OF MY PAPER WAS:—
TWO SEPARATE MECHANISMS EXIST
FOR UPSTREAM INFLUENCE VIA A BOUNDARY LAYER.
ARE THEY SPECIAL TO SUPERSONIC FLOW?
... WELL, YES AND NO!
MECHANISM (i) DOES ARISE (OSWATITSCH & WIEGHARDT 1941)
FROM A SPECIAL FEATURE OF SUPERSONIC
FLOW [AND SO FAILS IN SUBSONIC FLOW].
... MECHANISM (ii) DEPENDS HOWEVER
(LIEPMANN ROSHKO & DHAWAN 1949, LEES 1949, GADD & HOLDER 1952) ON UPSTREAM
SPREADING OF A SEPARATED-FLOW
REGION UNTIL IT IS SO SLENDER
AS TO CAUSE NO FURTHER SEPARATION.
BY COMPARING COMPREHENSIVE
EXPERIMENTS (MAIR 1952) ON SUCH
SPREADING, INVOLVING STEADY
AND UNSTEADY SEPARATED FLOWS [SUBJECT OF THIS COL-LOQUIUM]
WITH SOME ANALOGOUS SUBSONIC DATA,
I DEMONSTRATED VERY
BROAD SIMILARITIES [AND SOME INTERESTING MINOR DIFFERENCES].
IN THIS LECTURE I OUTLINE (i)
BRIEFLY, AND (ii) AT GREATER LENGTH;
OUTLINING SOURCES OF UNSTEADINESS.


THE OSWATITSCH & WIEGHARDT MECHANISM (i)
DEPENDS ON A SPECIAL FEATURE
OF FLOW AT MACH NUMBER $M_1 > 1$:
ITS DEFLEXION BY ANGLE η
CHANGES PRESSURE p_1 BY $A_2 \eta$,
WHERE $A_2 = \frac{\gamma p_1 M_1^2}{(M_1^2 - 1)^{1/2}}$; SO THAT (e.g.)
WALL CURVATURE $\frac{d\eta}{dx}$ GENERATES
PRESSURE GRADIENT $A_2 \frac{d\eta}{dx}$.
... SIMILARLY, CURVATURE $\frac{d^2\delta_1}{dx^2}$ OF
A BOUNDARY LAYER'S DISPLACEMENT
THICKNESS CONTOUR GIVES PRESSURE
GRADIENT $A_2 \frac{d^2\delta_1}{dx^2}$; WHICH IN TURN,
HOWEVER, MAY BE EXPECTED TO
THICKEN THE LAYER, AT A SPATIAL
RATE $\frac{d\delta_1}{dx} = A_1 \left(A_2 \frac{d^2\delta_1}{dx^2}\right)$ [WHERE A_1, THOUGH FAR FROM PRECISELY KNOWN, IS GREATER FOR LAMINAR THAN FOR TURBULENT LAYER].
THIS SUGGESTS e-FOLDING DISTANCE $A_1 A_2$.
AN ALTERNATIVE MECHANISM OF UPSTREAM
INFLUENCE 'PROPAGATION UP THE SUBSONIC
LAYER' [HOWARTH (1948); WITH VISCOSITY AFFECTING ONLY UNDISTURBED DISTRIBUTION, NOT DISTURBANCES]
HAD RUN INTO DIFFICULTIES. HOWEVER
RESOLUTION OF THESE (IN PART II OF MY PAPER)
EQUATED IT TO MECHANISM (i)
[WITH A_1 RELATIVELY PRECISELY DETERMINED].

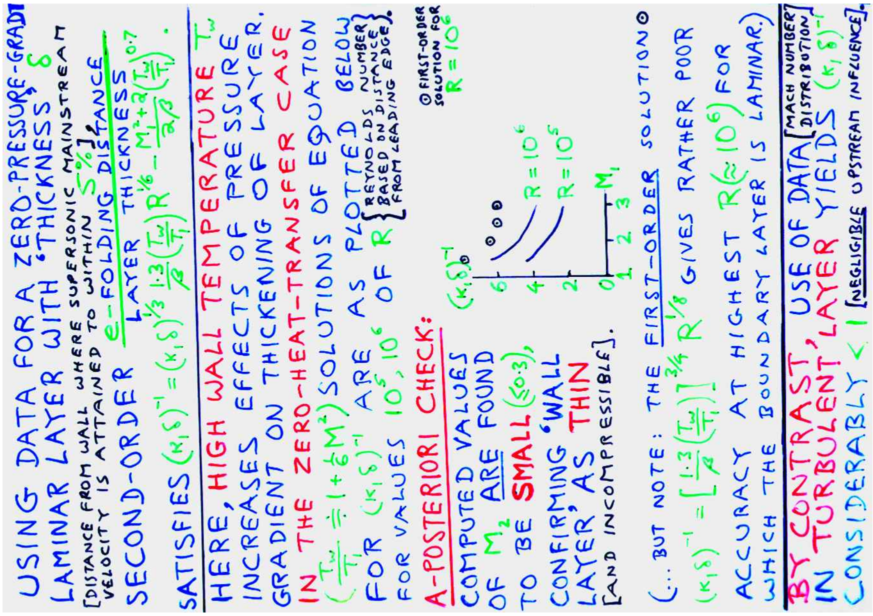

THE ESSENTIAL HINT: (FROM BOUNDARY-LAYER STABILITY THEORY)
SMALL DISTURBANCES $(u(y), v(y), 0)e^{ik(x-ct)}$
TO A PARALLEL FLOW $(U(y), 0, 0)$
SATISFY (AT LOW MACH NUMBER) THE ORR-SOMMERFELD
EQN. $[U(y)-c](v''-k^2v)-U''(y)v = \frac{\nu_w}{ik}(v''''-2k^2v''+k^4v)$
WHERE R.H.S. IS KNOWN TO SIGNIFY
BOTH (i) IN A WALL LAYER AROUND $U(y)=0$,
AND (ii) IN A CRITICAL LAYER AROUND $U(y)=c$.
FOR DISTURBANCES INDEPENDENT OF t,
$c=0$ AND THESE LAYERS COINCIDE.
ALSO, IF UNDISTURBED FLOW HAS ZERO
PRESSURE GRADIENT, $U''(0)=0$ AND SO
$U(y) \doteqdot U'(0)y$ (NEAR WALL). THEN $y(v''-k^2v)=L^3(v''''-2k^2v''+k^4v)$
WHERE $L = \left(\frac{\nu_w}{ikU'(0)}\right)^{1/3}$.
0·3 0·2 0·1 Ai(z) 1 2 3 z
SOLUTION FOR SMALL kL (LESS FINE-SCALE FEATURES, INCLUDING UPSTREAM INFLUENCE)
$yv''=L^3v''''$, GIVING $v''=A\,Ai\left(\frac{y}{L}\right)$ WITH
EXPONENTIAL DECAY OF DISTURBED {VISCOUS STRESS}.
ALSO $v = A\int_0^y (y-y_1)Ai\left(\frac{y_1}{L}\right)dy_1$, WHICH FOR $\frac{y}{L}$
LARGE (INVISCID REGION) $\sim A\left[yL\int_0^\infty Ai(z)dz - L^2\int_0^\infty z\,Ai(z)dz\right]$
$= A(\cdot 333yL - \cdot 259L^2)$.
THUS, INVISCID BEHAVIOUR HAS
v VANISHING WHERE $y = 0{\cdot}78L$, WITH
EFFECTIVE WALL MACH NUMBER $M_2 = 0{\cdot}78LM'(0)$
[COMPARE COMPUTED SOLUTIONS FOR GENERAL kL, GIVING INSIGNIFICANT CHANGE WHEN $|kL|<1$.]
kL 0 ±0·5 ±1·0 ±1·5 ±2·0
(y/L) 0·78 0·75 0·70 0·64 0·58


THIS CONCLUSION [ABOUT THE REQUIRED INVISCID SOLUTION BEING ONE WITH $v \to 0$ NOT WHERE $U(y)=0$ (WALL) BUT WHERE $U(y)=0{\cdot}78LU'(0)$ WITH MACH NUMBER $M_2=0{\cdot}78LM'(0)$; HERE, $L^3=\frac{\nu_w}{ikU'(0)}$]
IS JUST WHAT WE NEED! — BECAUSE
STEADY INVISCID DISTURBANCES TO A
PARALLEL FLOW WITH GENERAL MACH
NUMBER DISTRIBUTION $(M(y), 0, 0)$
SATISFY SIMPLE EQUATIONS IN
TERMS OF PRESSURE p AND DEFLEXION η:
$\frac{\partial p}{\partial x} = \gamma p_1 \frac{M^2(y)}{1-M^2(y)}\frac{\partial \eta}{\partial y}$, $\frac{\partial p}{\partial y} = -\gamma p_1 M^2(y)\frac{\partial \eta}{\partial x}$
[RELATING PRESSURE GRADIENTS TO STREAMTUBE-AREA VARIATION AND TO CENTRIFUGAL FORCE].
SOLUTION WITH $M(y)$ RANGING FROM
M_2 TO M_1 (FREE STREAM MACH NUMBER) IS NON-SINGULAR.
IF $\frac{p-p_1}{\gamma p_1} = \int_{-\infty}^{\infty} e^{ikx}\Pi(k,y)dk$, THEN Π SATISFIES
$\frac{d}{dy}\left(M^{-2}(y)\frac{d\Pi}{dy}\right) = k^2(M^{-2}(y)-1)\Pi$ {AND BOUNDARY CONDITIONS:-}
(i) IF WALL DEFLEXION [ZERO FOR A FLAT WALL]
IS $\eta = \int_{-\infty}^{\infty} e^{ikx}H(k)dk$, THEN $\Pi_y(k,0) = -M_2^2 ikH(k)$.
(ii) AT EDGE $y=\delta$ OF BOUNDARY LAYER
$\frac{p-p_1}{\gamma p_1} = f(x+\beta y)+g(x-\beta y) = \int_{-\infty}^{\infty} e^{ikx}[F(k)e^{ik\beta y}+G(k)e^{-ik\beta y}]dk$
WHERE $\beta = (M_1^2-1)^{1/2}$ AND f = INCIDENT WAVE [IF ANY]
GIVING $\Pi_y(k,\delta)+ik\beta\Pi(k,\delta) = 2ik\beta e^{ik\beta\delta}F(k)$.

USING TWO INDEPENDENT SOLUTIONS $\varphi(k,y)$, $T(k,y)$
OF $\frac{d}{dy}\left(M^{-2}(y)\frac{d\Pi}{dy}\right) = k^2\left(M^{-2}(y)-1\right)\Pi$ $\left\{\begin{matrix}\text{WITH } \varphi(k,0)=1, \varphi_y(k,0)=0 \\ \text{AND } T(k,0)=0, T_y(k,0)=1\end{matrix}\right\}$
THE SOLUTION WITH $\Pi_y(k,0) = -M_2^2 ikH(k)$ AND
WITH $\Pi_y(k,\delta)+ik\beta\Pi(k,\delta) = 2ik\beta e^{ik\beta\delta}F(k)$
IS $\Pi = 2ik\beta e^{ik\beta\delta}F(k)\dfrac{\varphi(k,y)}{\varphi_y(k,\delta)+ik\beta\varphi(k,\delta)}$ [FROM INCIDENT WAVE]
$-M_2^2 ikH(k)\left\{T(k,y)-\dfrac{T_y(k,\delta)+ik\beta T(k,\delta)}{\varphi_y(k,\delta)+ik\beta\varphi(k,\delta)}\right\}$ [FROM WALL DEFLEXION]
AND FOURIER TRANSFORM $G(k)$ OF EMITTED WAVE IS
$G(k) = \dfrac{-e^{2ik\beta\delta}F(k)[\varphi_y(k,\delta)-ik\beta\varphi(k,\delta)]+M_1^2 ik e^{ik\beta\delta}H(k)}{\varphi_y(k,\delta)+ik\beta\varphi(k,\delta)}$.

[FOR SMALL-SCALE EFFECTS, DEDUCED FROM WBK-LANGER LARGE-k SOLUTIONS, SEE BELOW; ON THE OTHER HAND,]
UPSTREAM INFLUENCE COMES FROM SMALL-k FORMS
$\varphi_y = k^2M^2(y)\int_0^y(M^{-2}(z)-1)dz$, $\varphi = 1+k^2\int_0^y M^2(t)dt\int_0^t(M^{-2}(z)-1)dz$.
AS $x\to-\infty$, ALL SOLUTIONS BEHAVE LIKE
$e^{\kappa_1 x}$ WITH κ_1 AS LEAST POSITIVE NUMBER
FOR WHICH $k=-i\kappa_1$ IS POLE $[\varphi_y(-i\kappa_1,\delta)+\kappa_1\beta\varphi(-i\kappa_1,\delta)=0]$.
(…NOTE THAT HERE $L=\left(\dfrac{2\nu_w}{\kappa_1 U'(0)}\right)^{1/3}$ IS REAL)

SMALL-k APPROXIMATION TO SECOND ORDER:
$-\kappa_1^2M_1^2\int_0^\delta(M^{-2}(z)-1)dz+\kappa_1\beta\left[1-\kappa_1^2\int_0^\delta M^2(t)dt\int_0^t(M^{-2}(z)-1)dz\right]=0$
GIVING TO FIRST ORDER $\int_0^\delta(M^{-2}(z)-1)dz = \dfrac{\beta}{\kappa_1 M_1^2}$
AND, TO SECOND, AN e-FOLDING DISTANCE
$\kappa_1^{-1} = \dfrac{M_1^2}{\beta}\int_0^\delta(M^{-2}(z)-1)dz + \dfrac{\beta}{M_1^2}\int_0^\delta M^2(z)dz$
[SHOWING NO VARIATION WITH CHOICE OF δ].
(SOLVE WITH $M(0)=M_2=M'(0)L$.)



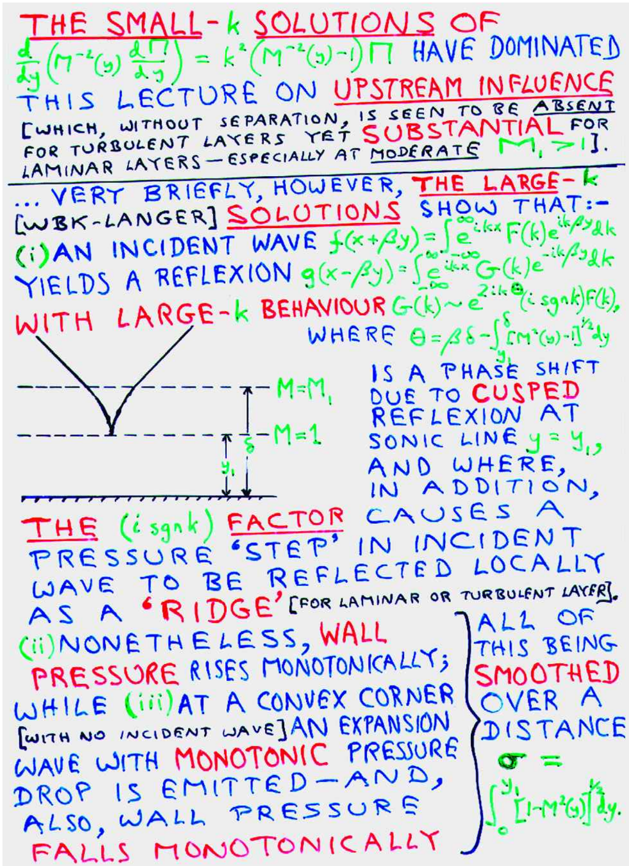
USING DATA FOR A ZERO-PRESSURE-GRADIENT LAMINAR LAYER WITH 'THICKNESS' δ
[DISTANCE FROM WALL WHERE SUPERSONIC MAINSTREAM VELOCITY IS ATTAINED TO WITHIN 5%],
SECOND-ORDER $\dfrac{e\text{-FOLDING DISTANCE}}{\text{LAYER THICKNESS}}$
SATISFIES $(\kappa_1\delta)^{-1} = (\kappa_1\delta)^{1/3}\dfrac{1.3}{\beta}\left(\dfrac{T_w}{T_1}\right)R^{1/6} - \dfrac{M_1^2+2}{2\beta}\left(\dfrac{T_w}{T_1}\right)^{0.7}$.

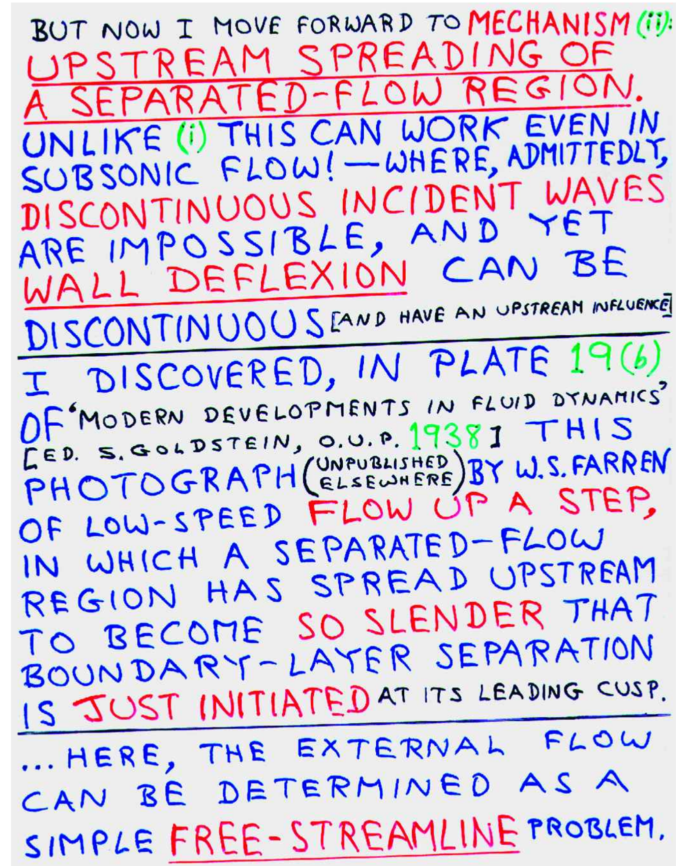
HERE, HIGH WALL TEMPERATURE T_w INCREASES EFFECTS OF PRESSURE GRADIENT ON THICKENING OF LAYER.
IN THE ZERO-HEAT-TRANSFER CASE $\left(\dfrac{T_w}{T_1}\doteqdot 1+\tfrac{1}{6}M^2\right)$ SOLUTIONS OF EQUATION
FOR $(\kappa_1\delta)^{-1}$ ARE AS PLOTTED BELOW
FOR VALUES $10^5, 10^6$ OF R {REYNOLDS NUMBER BASED ON DISTANCE FROM LEADING EDGE}.

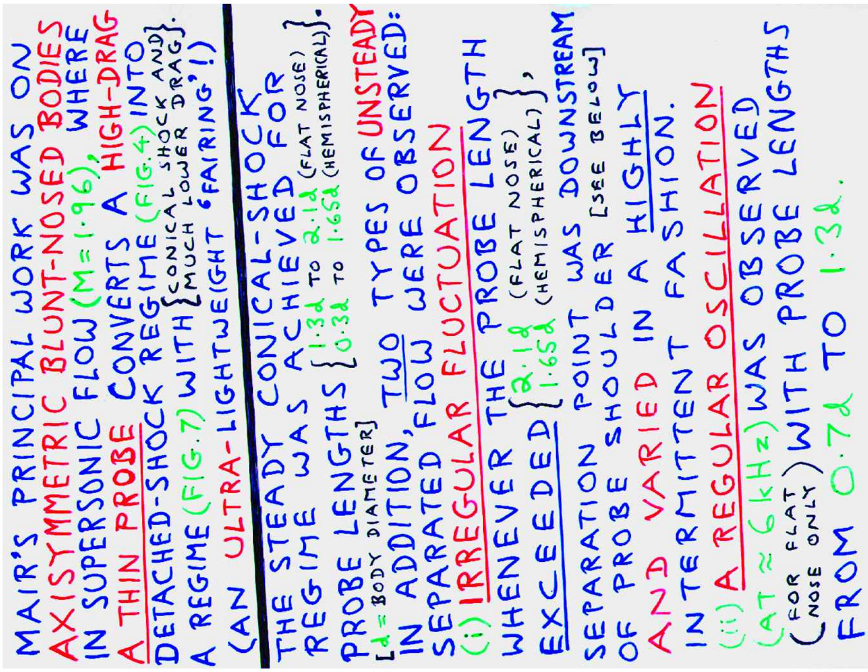
A-POSTERIORI CHECK:
COMPUTED VALUES OF M_2 ARE FOUND TO BE SMALL ($\lesssim 0.3$), CONFIRMING 'WALL LAYER' AS THIN [AND INCOMPRESSIBLE].

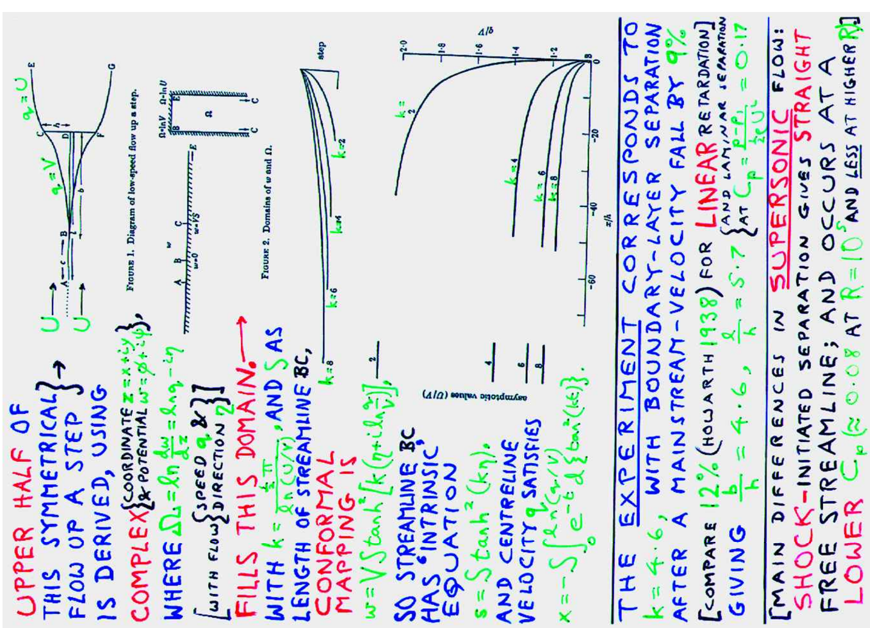
(… BUT NOTE: THE FIRST-ORDER SOLUTION ⊙
$(\kappa_1\delta)^{-1} = \left[\dfrac{1.3}{\beta}\left(\dfrac{T_w}{T_1}\right)\right]^{3/4}R^{1/8}$ GIVES RATHER POOR
ACCURACY AT HIGHEST $R(\approx 10^6)$ FOR WHICH THE BOUNDARY LAYER IS LAMINAR.)

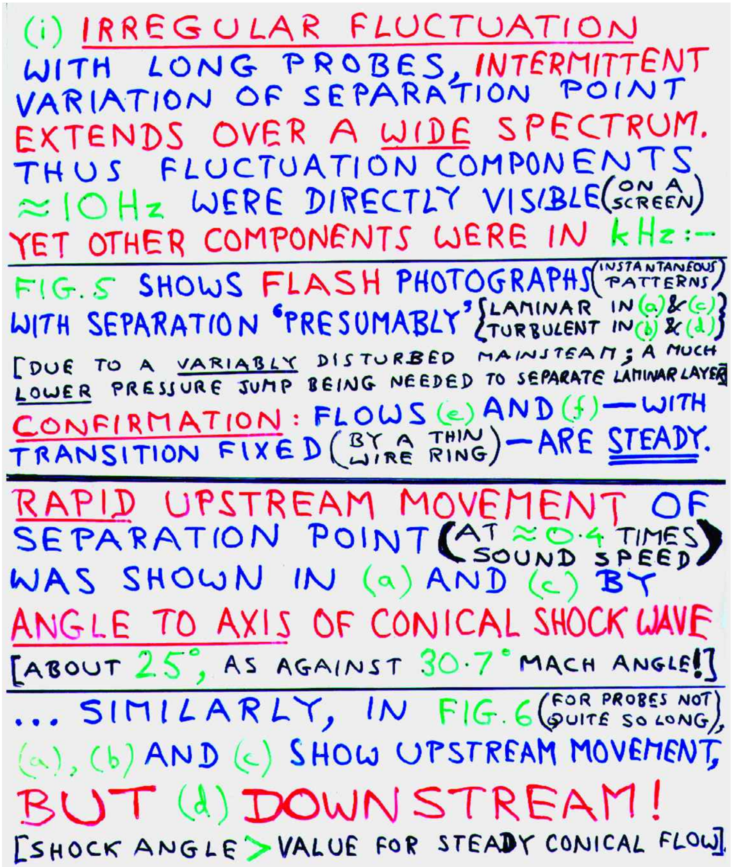
BY CONTRAST, USE OF DATA [MACH NUMBER DISTRIBUTION] IN TURBULENT LAYER YIELDS $(\kappa_1\delta)^{-1}$
CONSIDERABLY <1 [NEGLIGIBLE UPSTREAM INFLUENCE].


THE SMALL-k SOLUTIONS OF
$\frac{d}{dy}\left(M^{-2}(y)\frac{d\Pi}{dy}\right) = k^2\left(M^{-2}(y)-1\right)\Pi$ HAVE DOMINATED
THIS LECTURE ON UPSTREAM INFLUENCE
[WHICH, WITHOUT SEPARATION, IS SEEN TO BE ABSENT
FOR TURBULENT LAYERS YET SUBSTANTIAL FOR
LAMINAR LAYERS—ESPECIALLY AT MODERATE $M_1 > 1$].
... VERY BRIEFLY, HOWEVER, THE LARGE-k
[WBK-LANGER] SOLUTIONS SHOW THAT:-
(i) AN INCIDENT WAVE $f(x+\beta y) = \int_{-\infty}^{\infty} e^{ikx} F(k) e^{ik\beta y} dk$
YIELDS A REFLEXION $g(x-\beta y) = \int_{-\infty}^{\infty} e^{ikx} G(k) e^{-ik\beta y} dk$
WITH LARGE-k BEHAVIOUR $G(k) \sim e^{2ik\theta} (i\,\mathrm{sgn}\,k) F(k)$,
WHERE $\theta = \beta\delta - \int_{y_1}^{\delta} [M^2(y)-1]^{1/2} dy$
IS A PHASE SHIFT
DUE TO CUSPED
REFLEXION AT
SONIC LINE $y = y_1$,
AND WHERE,
IN ADDITION,
$M = M_1$
$M = 1$
δ
y_1
THE $(i\,\mathrm{sgn}\,k)$ FACTOR CAUSES A
PRESSURE 'STEP' IN INCIDENT
WAVE TO BE REFLECTED LOCALLY
AS A 'RIDGE' [FOR LAMINAR OR TURBULENT LAYER].
(ii) NONETHELESS, WALL
PRESSURE RISES MONOTONICALLY;
WHILE (iii) AT A CONVEX CORNER
[WITH NO INCIDENT WAVE] AN EXPANSION
WAVE WITH MONOTONIC PRESSURE
DROP IS EMITTED—AND,
ALSO, WALL PRESSURE
FALLS MONOTONICALLY
ALL OF
THIS BEING
SMOOTHED
OVER A
DISTANCE
$\sigma = \int_0^{y_1} [1-M^2(y)]^{1/2} dy$.


BUT NOW I MOVE FORWARD TO MECHANISM (ii):
UPSTREAM SPREADING OF
A SEPARATED-FLOW REGION.
UNLIKE (i) THIS CAN WORK EVEN IN
SUBSONIC FLOW!—WHERE, ADMITTEDLY,
DISCONTINUOUS INCIDENT WAVES
ARE IMPOSSIBLE, AND YET
WALL DEFLEXION CAN BE
DISCONTINUOUS [AND HAVE AN UPSTREAM INFLUENCE]
I DISCOVERED, IN PLATE 19(b)
OF 'MODERN DEVELOPMENTS IN FLUID DYNAMICS'
[ED. S. GOLDSTEIN, O.U.P. 1938] THIS
PHOTOGRAPH (UNPUBLISHED ELSEWHERE) BY W.S. FARREN
OF LOW-SPEED FLOW UP A STEP,
IN WHICH A SEPARATED-FLOW
REGION HAS SPREAD UPSTREAM
TO BECOME SO SLENDER THAT
BOUNDARY-LAYER SEPARATION
IS JUST INITIATED AT ITS LEADING CUSP.
... HERE, THE EXTERNAL FLOW
CAN BE DETERMINED AS A
SIMPLE FREE-STREAMLINE PROBLEM.


MAIR'S PRINCIPAL WORK WAS ON
AXISYMMETRIC BLUNT-NOSED BODIES
IN SUPERSONIC FLOW (M = 1·96), WHERE
A THIN PROBE CONVERTS A HIGH-DRAG
DETACHED-SHOCK REGIME (FIG. 4) INTO
A REGIME (FIG. 7) WITH {CONICAL SHOCK AND MUCH LOWER DRAG}.
(AN ULTRA-LIGHTWEIGHT 'FAIRING'!)
THE STEADY CONICAL-SHOCK
REGIME WAS ACHIEVED FOR
PROBE LENGTHS {1·3d TO 2·1d (FLAT NOSE) / 0·3d TO 1·65d (HEMISPHERICAL)}.
[d = BODY DIAMETER]
IN ADDITION, TWO TYPES OF UNSTEADY
SEPARATED FLOW WERE OBSERVED:
(i) IRREGULAR FLUCTUATION
WHENEVER THE PROBE LENGTH
EXCEEDED {2·1d (FLAT NOSE) / 1·65d (HEMISPHERICAL)},
SEPARATION POINT WAS DOWNSTREAM
OF PROBE SHOULDER [SEE BELOW]
AND VARIED IN A HIGHLY
INTERMITTENT FASHION.
(ii) A REGULAR OSCILLATION
(AT ≈ 6 kHz) WAS OBSERVED
(FOR FLAT NOSE ONLY) WITH PROBE LENGTHS
FROM 0·7d TO 1·3d.


UPPER HALF OF
THIS SYMMETRICAL FLOW UP A STEP
IS DERIVED, USING
COMPLEX {COORDINATE z = x + iy & POTENTIAL w = φ + iψ},
WHERE Ω = ln dw/dz = ln q − iη
[WITH FLOW {SPEED q & DIRECTION η}]
FILLS THIS DOMAIN.
WITH k = ½π / ln(U/V), AND S AS
LENGTH OF STREAMLINE BC,
CONFORMAL
MAPPING IS
w = V S tanh²[k(η + i ln q/V)],
SO STREAMLINE BC
HAS 'INTRINSIC'
EQUATION
s = S tanh²(kη).
AND CENTRELINE
VELOCITY q SATISFIES
x = −S ∫₀^{ln(q/V)} e^{−t} d{tanh²(kt)}.
FIGURE 1. Diagram of low-speed flow up a step.
FIGURE 2. Domains of w and Ω.
step
asymptotic values (U/V)
z/h
Δ/δ
k = 2
k = 4
k = 6
k = 8
THE EXPERIMENT CORRESPONDS TO
k = 4·6, WITH BOUNDARY-LAYER SEPARATION
AFTER A MAINSTREAM-VELOCITY FALL BY 9%
[COMPARE 12% (HOWARTH 1938) FOR LINEAR RETARDATION]
GIVING b/h = 4·6, l/h = 5·7 {AND LAMINAR SEPARATION AT C_p = (p − p₁)/(½ρU²) = 0·17}
[MAIN DIFFERENCES IN SUPERSONIC FLOW:
SHOCK-INITIATED SEPARATION GIVES STRAIGHT
FREE STREAMLINE; AND OCCURS AT A
LOWER C_p (≈ 0·08 AT R = 10⁵ AND LESS AT HIGHER R)]


(i) IRREGULAR FLUCTUATION
WITH LONG PROBES, INTERMITTENT
VARIATION OF SEPARATION POINT
EXTENDS OVER A WIDE SPECTRUM.
THUS FLUCTUATION COMPONENTS
≈10Hz WERE DIRECTLY VISIBLE (ON A SCREEN)
YET OTHER COMPONENTS WERE IN kHz:—
FIG.5 SHOWS FLASH PHOTOGRAPHS (INSTANTANEOUS PATTERNS)
WITH SEPARATION 'PRESUMABLY' {LAMINAR IN (a) & (c) / TURBULENT IN (b) & (d)}
[DUE TO A VARIABLY DISTURBED MAINSTEAM; A MUCH
LOWER PRESSURE JUMP BEING NEEDED TO SEPARATE LAMINAR LAYER]
CONFIRMATION: FLOWS (e) AND (f) — WITH
TRANSITION FIXED (BY A THIN WIRE RING) — ARE STEADY.
RAPID UPSTREAM MOVEMENT OF
SEPARATION POINT (AT ≈0·4 TIMES SOUND SPEED)
WAS SHOWN IN (a) AND (c) BY
ANGLE TO AXIS OF CONICAL SHOCK WAVE
[ABOUT 25°, AS AGAINST 30·7° MACH ANGLE!]
... SIMILARLY, IN FIG.6 (FOR PROBES NOT QUITE SO LONG),
(a), (b) AND (c) SHOW UPSTREAM MOVEMENT,
BUT (d) DOWNSTREAM!
[SHOCK ANGLE > VALUE FOR STEADY CONICAL FLOW]


AND MAIR FOUND
(ii) REGULAR OSCILLATION (AT ≈ 6kHz)
ON FLAT-NOSED BODY WITH PROBE
LENGTHS FROM 0·7d TO 1·3d;
AND IDENTIFIED ITS NATURE BY
FIRST TAKING NUMEROUS FLASH PHOTOS
AND THEN SEQUENCING A SELECTION:
FIG.12 (a) WITH 2 BOW WAVES [UPSTREAM WAVE MOVING AFT, TO YIELD IN (f) A SINGLE 'SPLIT' BOW WAVE]
WHILE SEPARATION MOVES UPSTREAM;
(b) 50μs LATER, WITH SEPARATION
AT PROBE NOSE GIVING {LARGE DEAD-AIR REGION & INCIPIENT STRONG SHOCK};
(c) 30μs LATER, WITH BIGGER DEAD-AIR
REGION, WITH PROBE BOW SHOCK EXTENDED
AND WITH ANNULAR BODY BOW WAVE;
(d) 20μs LATER, WITH ANNULAR WAVE
MOVING AFT [ORIGIN OF EMISSION OF WEAK DOWNSTREAM-MOVING SHOCKS];
(e) 40μs LATER, AND
(f) 20μs LATER STILL } WITH DEAD-AIR REGION CONTRACTING, AND ITS SHOCK MOVING AFT, WHILE NEW BODY BOW WAVE
APPEARS — TO GIVE (a) SOON AFTER!
... FINALLY, SOME CORRESPONDING 2D FLOWS:
FIG.15 (a) THICK PLATE, NO THIN PLATE;
(b), (c) THIN PLATES OF LENGTH 0·55d (SAME BOW WAVE)
AND 2·5d [STEADY WEDGE-SHAPED DEAD-AIR REGION];
(d), (e), (f) WITH INTERMEDIATE LENGTHS
1·5d AND 2·0d UNSTEADY {FLOWS; NEED NOT BE SYMMETRICAL!}
CONCLUSION: 45 YEARS AGO
'B.L.&U.I' WAS ALIVE AND WELL AND
LIVING IN MANCHESTER! (INTER ALIA)